Numerical answers

Electricity and magnetism

1 Coulomb's law

1.1A $\mathbf{F} = -8.24 \times 10^{-8}\ \text{N}\ \hat{\mathbf{r}}$

1.2A (i) $\mathbf{F} = -1.81 \times 10^{-10}\ \text{N}\ \hat{\mathbf{r}}$

(ii) $\mathbf{F} = -2.32 \times 10^{-12}\ \text{N}\ \hat{\mathbf{r}}$

1.3A $\mathbf{F} = +7.4 \times 10^{-8}\ \text{N}\ \hat{\mathbf{r}}$

1.4A $r = 7.20 \times 10^{-10}\ \text{m}$

1.5B (i) (0,0,0): $\mathbf{F} = \dfrac{2qe}{\pi\varepsilon d^2}\begin{pmatrix}1\\0\\0\end{pmatrix}$

(ii) (–a,0,0): $\mathbf{F} = \dfrac{-qe}{4\pi\varepsilon\left(a^2 - ad + d^2/4\right)^{1/2}}\begin{pmatrix}1\\0\\0\end{pmatrix} + \dfrac{qe}{4\pi\varepsilon\left(a^2 + ad + d^2/4\right)^{1/2}}\begin{pmatrix}1\\0\\0\end{pmatrix}$

(iii) (a,0,0): by symmetry from (ii)

$$\mathbf{F} = \frac{qe}{4\pi\varepsilon\left(a^2 + ad + d^2/4\right)^{1/2}}\begin{pmatrix}1\\0\\0\end{pmatrix} - \frac{qe}{4\pi\varepsilon\left(a^2 - ad + d^2/4\right)^{1/2}}\begin{pmatrix}1\\0\\0\end{pmatrix}$$

(iv) (0,a,0): $\mathbf{F} = \dfrac{qed}{4\pi\varepsilon\left(a^2 + d^2/4\right)^{3/2}}\begin{pmatrix}1\\0\\0\end{pmatrix}$

(v) (0,–a,0): $\mathbf{F} = \dfrac{qed}{4\pi\varepsilon\left(a^2 + d^2/4\right)^{3/2}}\begin{pmatrix}1\\0\\0\end{pmatrix}$

(vi) (a,a,a):

$$\mathbf{F} = \frac{qe}{4\pi\varepsilon\left(3a^2 + ad + d^2/4\right)^{3/2}}\begin{pmatrix}a + d/2\\a\\a\end{pmatrix} - \frac{qe}{4\pi\varepsilon\left(3a^2 - ad + d^2/4\right)^{3/2}}\begin{pmatrix}a - d/2\\a\\a\end{pmatrix}$$

1.6B Maximum repulsion is obtained when division is equal.

1.7B (a) The resultant force on the central particle is $\frac{q_1 q_2}{4\pi\varepsilon r_{12}^2} - \frac{q_3 q_2}{4\pi\varepsilon r_{32}^2}$ in the x direction. At equilibrium this force is zero.

(b) $q_1 (r-x)^2 = q_3 x^2$, hence $\frac{r-x}{x} = \sqrt{\frac{q_3}{q_1}} \Rightarrow x = \frac{r}{1+\sqrt{q_3/q_1}}$

(c) By Newton's first law $\mathbf{F}_{31} = -\mathbf{F}_{13}$ and the criterion for equilibrium is $\mathbf{F}_{21} + \mathbf{F}_{23} = 0$. Hence $\mathbf{F}_{12} = -\mathbf{F}_{32}$. Formally $\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} = -\mathbf{F}_{32} - \mathbf{F}_{31} = -\mathbf{F}_3$.

1.8B (a) The resultant force on particle 3 is $\frac{q_3 q_1}{4\pi\varepsilon r_{31}^2} + \frac{q_3 q_2}{4\pi\varepsilon r_{32}^2}$ in the x direction. At equilibrium this force is zero. There can be no equilibrium position between 0 and r as 3 would be attracted and pushed towards 0.

(b) Hence $q_1 (x-r)^2 = -q_2 x^2 = |q_2| x^2 \Rightarrow \frac{|q_2|}{q_1} = \frac{(x-r)^2}{x^2} = \left(1 - \frac{r}{x}\right)^2$.

$$x = \frac{r}{1-\sqrt{|q_2|/q_1}}.$$

1.9S $F = -\frac{8q^2 \delta x}{\pi\varepsilon r^3} + \ldots$

2 Electric field

2.1S (i) $\mathbf{E} = +5.15\times10^{11}\ \mathrm{N\,C^{-1}}\ \hat{\mathbf{r}}$

(ii) $\mathbf{E} = +1.13\times10^{9}\ \mathrm{N\,C^{-1}}\ \hat{\mathbf{r}}$ in vacuum, $+1.45\times10^{7}\ \mathrm{N\,C^{-1}}\ \hat{\mathbf{r}}$ in water

(iii) $\mathbf{E} = +4.61\times10^{11}\ \mathrm{N\,C^{-1}}\ \hat{\mathbf{r}}$

(iv) $\mathbf{E} = -2.78\times10^{9}\ \mathrm{N\,C^{-1}}\ \hat{\mathbf{r}}$

(v) $\mathbf{E} = \frac{2e}{\pi\varepsilon d^2}\begin{pmatrix}1\\0\\0\end{pmatrix}$

$$\mathbf{E} = \frac{-e}{4\pi\varepsilon\left(a^2 - ad + d^2/4\right)^{1/2}}\begin{pmatrix}1\\0\\0\end{pmatrix} + \frac{e}{4\pi\varepsilon\left(a^2 + ad + d^2/4\right)^{1/2}}\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\mathbf{E} = \frac{e}{4\pi\varepsilon\left(a^2 + ad + d^2/4\right)^{1/2}}\begin{pmatrix}1\\0\\0\end{pmatrix} - \frac{e}{4\pi\varepsilon\left(a^2 - ad + d^2/4\right)^{1/2}}\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\mathbf{E} = \frac{ed}{4\pi\varepsilon\left(a^2 + d^2/4\right)^{3/2}}\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\mathbf{E} = \frac{ed}{4\pi\varepsilon\left(a^2 + d^2/4\right)^{3/2}}\begin{pmatrix}1\\0\\0\end{pmatrix}$$

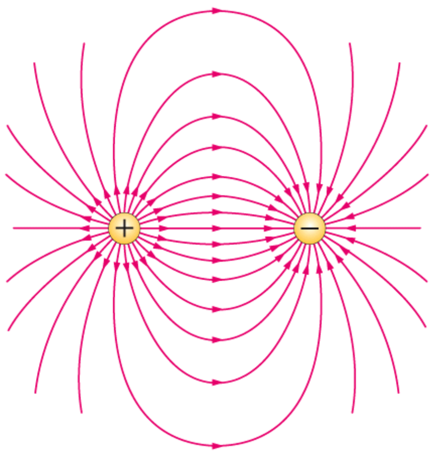
$$\mathbf{E} = \frac{e}{4\pi\varepsilon\left(3a^2 + ad + d^2/4\right)^{3/2}}\begin{pmatrix}a+d/2\\a\\a\end{pmatrix} - \frac{e}{4\pi\varepsilon\left(3a^2 - ad + d^2/4\right)^{3/2}}\begin{pmatrix}a-d/2\\a\\a\end{pmatrix}$$

2.2B (i) $-\frac{0.31e}{4\pi\varepsilon r^2}\hat{\mathbf{i}} - \frac{0.31e}{4\pi\varepsilon r^2}\left(-\hat{\mathbf{i}}\right) = \mathbf{0}$.

(ii) $-\frac{0.15e}{4\pi\varepsilon r^2}\left(-\hat{\mathbf{i}}\right) - \frac{0.15e}{4\pi\varepsilon r^2}\left(\hat{\mathbf{i}}/2 - \hat{\mathbf{j}}\sqrt{3/2}\right) - \frac{0.15e}{4\pi\varepsilon r^2}\left(\hat{\mathbf{i}}/2 + \hat{\mathbf{j}}\sqrt{3/2}\right) = \mathbf{0}$.

2.3B $\mathbf{E} = \begin{pmatrix}0\\0\\-1.122\end{pmatrix}\times 10^8 \text{ N C}^{-1}$

2.5A



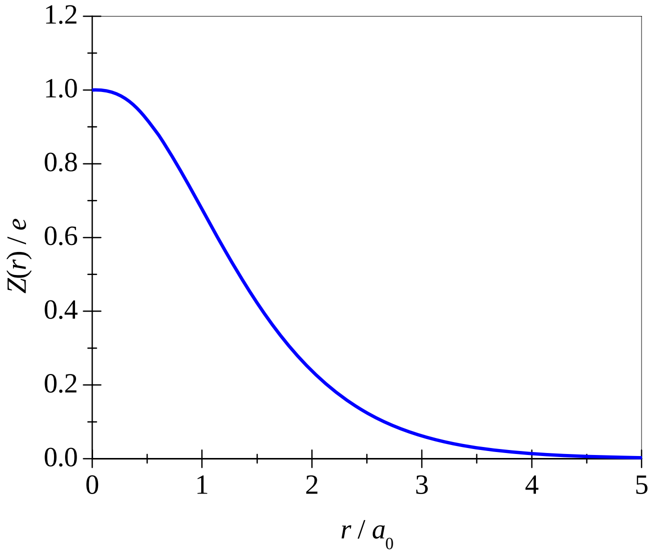
2.6B The equilibrium position is where the force on a test charge is equal to zero, this can only happen at a location where the electric field is zero, and applies to any value of the test charge.

2.7C (i) $\int_0^\infty 4\pi r^2 \rho\, dr = \int_0^\infty 4r^2 \frac{\exp(-2r/a_0)}{a_0^3} dr = \left[-\left(1 + \frac{2r}{a_0} + \frac{2r^2}{a_0^2}\right)\exp(-2r/a_0)\right]_0^\infty = 1$

(ii) The enclosed charge is the nuclear charge plus the part of the electron charge inside the sphere. The field is normal to the sphere. Hence

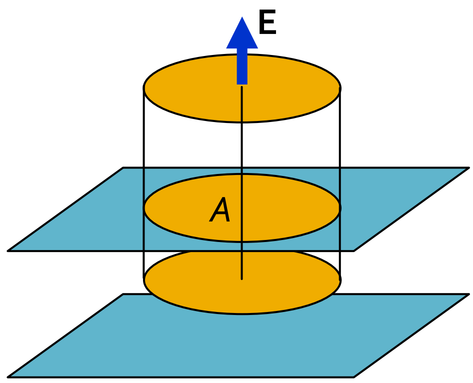
$EA = 4\pi r^2 E\varepsilon_0 = e - e\int_0^r 4\pi r^2 \rho\, dr$, hence $E = \frac{e}{4\pi\varepsilon_0 r^2}\left(1 + \frac{2r}{a_0} + \frac{2r^2}{a_0^2}\right)\exp(-2r/a_0)$

(iii) $eZ(r) = e\left(1 + \frac{2r}{a_0} + \frac{2r^2}{a_0^2}\right)\exp(-2r/a_0)$.



2.8C (i) The electric field is perpendicular to the plane of the electrode. Inside the electrode there is no enclosed charge (because charges are concentrated at the surface) and so the field is zero.

(ii) Construct a cylinder perpendicular to the electrode, one of whose faces is enclosed in the electrode, and the other is in the solution



On the face inside the electrode $E = 0$. On the cylindrical sides E is parallel to the surface so the electric flux is zero, for the face in the solution $EA = q/\varepsilon$, where q is the charge enclosed within the cylinder and ε is the permittivity of the solution, $\varepsilon = \varepsilon_0\varepsilon_r$. If the top of the cylinder is a distance z from the electrode, the enclosed charge is $q = \sigma A - \sigma A\int_0^x \kappa\exp(-\kappa x')dx' = \sigma A\exp(-\kappa x)$.

Hence $E = \frac{q}{\varepsilon A} = \frac{\sigma \exp(-\kappa x)}{\varepsilon}$, operating in the *x* direction.

3 Potential energy and potential

3.1A (a) $U_{el} = \frac{z_1 z_2 e^2}{4\pi\varepsilon_0\varepsilon_r r}$

(b) (i) 0.72 nm (ii) 29.7 nm

In order to react the ions must approach to within a few Angstroms of one another. In water the coulomb barrier to this approach will be of the same order as *kT*, and the approach is feasible, but in hexane the barrier will be much larger, of the order of 100 *kT*, and it is extremely unlikely that any such reaction will take place.

3.2A (a) 174 kJ mol^{-1} = 2.89×10^{-19} J

(b) *r* = 7.98 Å

3.3A (i) $U_{el} = = -1.54\times10^{-19}$ J

(ii) $U_{el} = = -1.91\times10^{-21}$ J

3.4A (i) nearest neighbours – 5.01×10^{-18} J

(ii) next nearest + 7.10×10^{-18} J

(iii) corners – 3.86×10^{-18} J

The sum of these contributions is – 1.78×10^{-18} J.

3.5B (i) $E =$ 20 kV m^{-1}

(ii) $F = -3.2\times10^{-15}$ N

(iii) $t =$ 2.07 ns.

3.6A 1 eV = 1.602×10^{-19} J

3.7B (i) KE = 1.60×10^{-15} J, *v* = 3.47×10^{5} m s^{-1}.

(ii) KE = 1.60×10^{-15} J, *v* = 1.57×10^{5} m s^{-1}.

3.8B (i) $E =$ 20 kV m^{-1}

(ii) $F =$ 8.0×10^{-15} N

(iii) $C =$ 2.1×10^{-10} F, $Q =$ 2.1×10^{-7} C.

3.9C (a) (i) $E = \lambda / 2\pi\varepsilon_0\varepsilon_r r$. (ii) E = 0.

(b) $\Delta V = -\dfrac{\lambda}{2\pi\varepsilon_0\varepsilon_r}\ln\dfrac{r_2}{r_1}$, capacitance per unit length is $\dfrac{2\pi\varepsilon_0\varepsilon_r}{\ln\dfrac{r_2}{r_1}}$.

4 Millikan experiment

4.1A (i) $r =$ 77.6 nm.

(ii) $\sigma = 2.12$ nC m^{-2}.

4.2A $\Delta V =$ 2.0 kV.

4.3B (i) $m = 8.17\times10^{-14}$ kg.

(ii) At equilibrium $q = -10e$.

4.4C In Millikan's experiment two consecutive measurements were as follows with a drop distance of 10.21 mm. The drop (drop 14) was timed at 18.804 s in its fall, and then when a potential difference of 5075 V was switched on between the plates separated by 16 mm, it rose through the same distance in 65.416 s. In the next measurement with the same drop the fall time was 18.662 s and the rise time 118.97 s. Calculate the charge on the drop in the two experiments and the difference between these charges. You may use the following data: T = 296.25 K, p = 100.4 kPa, $\eta = 1.825\times10^{-5}$ kg m^{-1} s^{-1}, g = 9.80 m s^{-2}, ρ_{oil} = 919.9 kg m^{-3}, ρ_{air} = 1.2 kg m^{-3}, $b = 7.88\times10^{-3}$ Pa m.

This is a sample of Millikan's data.

(i) Calculate the radius of the drop. In the absence of the field

$m'g = \dfrac{6\pi\eta a v_g}{1 + b/pa}$ accounting for the first order correction for Stokes' law and

$m' = \dfrac{4}{3}\pi a^3(\rho_{oil} - \rho_{air})$, accounting for the buoyancy due to displaced air.

a = 2.187 μm for the first experiment and 2.195 μm for the second.

(ii) Charges are $q = 1.60\times10^{-18}$ C and $q = 1.45\times10^{-18}$ C, difference of 1.5×10^{-19} C. [Using the complete set of data for drop 14 the best value of the electronic charge is 1.60×10^{-19} C.]

5 Dipole moments

5.1A T shaped. Underlying structure approximately trigonal bipyramid with lone pairs on the equator.

5.2B (i) Charges are $\pm e$. $p = 5.77\times10^{-29}$ C m = 1.73 D

(ii) The PE of interaction of these charges is 6.41×10^{-19} J.

5.3B (a) For a neutral molecule $\mathbf{p} = \sum_i q_i \mathbf{r}_i$

(b) Assume the charge on each O is $-q$, and on S is $+2q$.
$q = 3.81\times10^{-20}\ \mathrm{C} = 0.238e$.

5.4A $q = 7.38\times10^{-20}$ C = 0.461e. Hence 46.1% ionic character.

5.5A The energy is minimum when **p** and **r** are parallel, i.e. the positive end of the dipole points towards the anion.

$U_{el} = \dfrac{qp}{4\pi\varepsilon_0\varepsilon_r r^2}$ $= -1.08\times10^{-22}$ J.

Thermal energy at 298 K is 6.07×10^{-21} J, indicating that there is no strong preference for orientation at this distance in water at 293 K.

5.6B (a) $\mathbf{E} = \begin{pmatrix} 0 \\ 0 \\ -9.63 \end{pmatrix} \times 10^7\ \mathrm{N\ C^{-1}}$

(b) The dipole moment is 1.47 D in the negative direction. (For water the dipole moment is 1.85 D in the negative z direction.)

For ammonia $\mathbf{E} = \begin{pmatrix} 0 \\ 0 \\ -8.82 \end{pmatrix} \times 10^7\ \mathrm{V\ m^{-1}}$. (For water $\mathbf{E} = \begin{pmatrix} 0 \\ 0 \\ -1.11 \end{pmatrix} \times 10^8\ \mathrm{V\ m^{-1}}$.)

Both of these are close to the precise values.

5.7B The interaction energy is –**E.p**.
If **E** is in the z direction this is $U = -\mathbf{E.p} = -Ep\cos\theta$.

5.8B The force on each charge is $q\mathbf{E}$. This resolves into $qE\cos\theta$ for the stretching force $qE\sin\theta$ for the twisting component.